Initial Engineering Calculations

Flat Belt Tire Tester — MECH 493 Senior Design — Kettering University

This section presents the hand calculations used to size and verify each major subsystem of the flat belt tire testing machine. These analytical results established the initial design and component selections; the finite element analysis (FEA) results that follow validate and refine these calculations.

1. Design Inputs and Assumptions

Parameter	Value	Basis
Maximum belt speed	25 mph (36.7 ft/s)	GEM e2 top speed
Nominal tire load	550 lbf	GEM corner weight + dynamic transfer
Maximum tire load	900 lbf	Linear actuator stall force
Tire diameter	21.1 in	GEM stock tire
Belt width	16 in	Treadmill belt stock
Belt-deck friction (mu)	0.12	Waxed treadmill belt on UHMW
Tire rolling resistance (Crr)	0.015	Typical passenger tire
Drivetrain efficiency	0.93	Chain + bearings + coupling
Steel yield (A36 / 1018)	36,000 psi	Material property
Aluminum yield (6061-T6)	40,000 psi	Material property

2. Motor Power Sizing

2.1 Resistive force at the belt (at 550 lbf nominal load)

Belt-deck friction:

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F_deck = mu x W = 0.12 x 550 = 66.0 lbf
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Rolling resistance:

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F_rr = Crr x W = 0.015 x 550 = 8.25 lbf
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Bearing and belt-flex losses (estimated): F_misc = 11 lbf

Total: F_total = 66.0 + 8.25 + 11 = 85.25 lbf

2.2 Power required

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P_belt = F_total x v = 85.25 lbf x 36.7 ft/s = 3,126 ft-lbf/s
P_belt = 3,126 / 550 = 5.68 HP
P_motor = P_belt / efficiency = 5.68 / 0.93 = 6.11 HP
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Required motor power at 550 lbf, 25 mph = 6.11 HP

2.3 Power at other operating points

Tire Load	Total Force	Motor Power	% of 5 HP rated
360 lbf (typical)	59.6 lbf	4.27 HP	85%
550 lbf (nominal)	85.25 lbf	6.11 HP	122%
900 lbf (max)	132.5 lbf	9.50 HP	190%

Motor selected: 5 HP (NAE PE184T-5-4-ODP) with Service Factor 1.25. Sized for the typical 360 lbf test condition with thermal margin; higher loads reduce top speed (see operating envelope).

Note: these hand calculations cover the static and steady-running load cases. The braking load case, which adds a horizontal traction force at the tire contact patch (up to mu x load), is addressed in the FEA section because it produces combined (multi-axis) loading best evaluated numerically.

3. Drivetrain and Belt Speed

3.1 Required roller speed for 25 mph

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v = 25 mph = 440 in/s. Roller circumference = pi x 3.5 = 11.0 in
N_roller = (440 x 60) / 11.0 = 2,400 RPM required
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3.2 Sprocket ratio (step-up)

Motor 1,800 RPM is slower than the 2,400 RPM needed, so a step-UP is required.

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Ratio = 45T (motor) / 32T (roller) = 1.41:1 step-up
N_roller = 1,800 x (45/32) = 2,531 RPM
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Belt speed = 2,531 x 11.0 / 60 = 463 in/s = 26.4 mph (VFD trims to 25.0)

3.3 Chain force

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Motor continuous torque: T = 5 HP x 5252 / 1800 x 12 = 175 lb-in
45T #35 sprocket pitch diameter = 0.375 / sin(180/45) = 5.39 in
Chain force (peak) = T_peak / (PD/2); peak ~ 522 lb-in -> F = 194 lbf
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Chain FoS = 480 lbf (working load) / 194 = 2.5

Note: peak torque of 522 lb-in is used as a conservative value for chain sizing. A NEMA Design B motor's breakdown torque is typically about 2.75x continuous (approximately 481 lb-in here); the slightly higher 522 lb-in gives additional margin. Under normal running torque (111 lb-in) the chain force and FoS are far more favorable.

3.4 Drivetrain torque check

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Roller torque at 550 lbf load: T = 85 lbf x 1.75 in = 149 lb-in
Motor torque required = 149 / (1.41 x 0.95) = 111 lb-in
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Torque FoS = 175 / 111 = 1.58

4. Drive Roller Assembly

4.1 Roller tube bending (6061-T6, 3.5 OD x 0.1875 wall)

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Belt tension load (capstan): T1 + T2 = 594 lbf across 16 in belt zone
Section modulus S = 1.534 in^3; Max moment M = 1,634 lb-in
Stress = M / S = 1,065 psi
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FoS = 40,000 / 1,065 = 37.6; Deflection = 0.0017 in

Note: belt tension (T1 + T2) is found from the capstan (Euler belt-friction) equation using the peak drive force, a lagged-rubber friction coefficient of 0.30, and 180 degrees of belt wrap. This perpendicular belt load is distributed across the 16 in belt contact zone and treated as a partial uniform load on the roller spanning 19 in between bearings.

4.2 Dead axle shaft (3/4 in, 1018 steel, bearings 0.25 in inboard)

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Load per bearing = 594 / 2 = 297 lbf; Moment arm = 0.25 in
Max moment M = 297 x 0.25 = 74 lb-in; S = 0.041 in^3
Stress = 74 / 0.041 = 1,793 psi
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FoS = 36,000 / 1,793 = 20.1; Deflection = 0.0074 in

4.3 Roller bearings (R12-2RS sealed ball bearing)

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Operating load 297 lbf; dynamic rating 1,630 lbf
Operating speed 2,531 RPM; max rated 8,500 RPM
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Load FoS = 1,630 / 297 = 5.5; Speed FoS = 8,500 / 2,531 = 3.4

L10 life (dead-axle derated) approx 815 hours (~8 years lab use)

5. Belt Support Deck

Construction: 1/2 in UHMW-PE on 6 x 2x2x1/4 steel tubes butted together (12 in continuous support).

5.1 UHMW under tire (compression only — continuous support beneath)

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Contact pressure = 900 lbf / (4 x 6 in) = 37.5 psi
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FoS = 3,500 psi (UHMW) / 37.5 = 93

5.2 Steel cross tubes (2x2x1/4, 26 in span)

```
Realistic case (load shared by 3 tubes under patch): 300 lbf/tube
Stress = 1,975 psi -> FoS = 18.2, deflection 0.004 in
Worst case (single tube takes 900 lbf): stress 6,418 psi
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FoS = 5.6 (worst case), deflection 0.013 in

Note: because the six tubes are butted with no gaps, the UHMW sheet has continuous steel support and carries only compression (no bending spans). The tire contact patch (about 6 in long) sits over roughly three tubes, so the realistic case shares the load across three tubes. The single-tube case is a conservative bound assuming one tube carries the entire 900 lbf.

6. Tire Loading System

Note: the loading mechanism is being revised from a single lever to a 4-bar parallelogram linkage to eliminate camber change. Calculations for both the single-lever (initial) and the 4-bar arms are shown.

6.1 Lever arm bending (single-lever, 2x3x1/4 tube, 19 in)

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Max moment M = 900 lbf x 19 in = 17,100 lb-in; S = 1.698 in^3
Stress = 17,100 / 1.698 = 10,071 psi
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FoS = 36,000 / 10,071 = 3.6 (vertical load only)

6.2 4-bar arm axial load (2x2x1/4 tube)

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Axial force per arm = W x offset / separation = 900 x 4 / 8 = 450 lbf
Axial stress over 1.75 in^2 area = 257 psi -> FoS >100
Buckling (19 in, pin-pin): P_cr >> applied load, FoS >1,000
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4-bar arms carry mostly axial load; 2x2x1/4 is more than adequate

6.3 Pivot shaft (1 in dia, 1018 steel, 3.25 in bushing span)

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Worst-case load = actuator + belt reaction = 1,800 lbf
Max moment M = 1,800 x 3.25 / 4 = 1,462 lb-in; S = 0.098 in^3
Stress = 1,462 / 0.098 = 14,897 psi
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FoS = 36,000 / 14,897 = 2.4; Deflection = 0.0009 in

Note: the 1,800 lbf worst-case pivot load is the sum of the actuator force (900 lbf at stall) and the equal-and-opposite belt reaction (900 lbf) in the 1st-class single-lever configuration, where both act on the same side of the pivot. The 4-bar linkage reduces this pivot load substantially because loads are shared across four joints; the 1,800 lbf value is therefore conservative for the revised design.

6.4 Pivot bushings (841 bronze flanged sleeve, 1 in bore x 0.75 in)

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Load per bushing (worst case) = 900 lbf; projected area = 1.0 x 0.75 = 0.75 in^2
Pressure = 900 / 0.75 = 1,200 psi (rating 2,000 psi)
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FoS = 2,000 / 1,200 = 1.67 (worst case); 2.7 at nominal load

7. Actuator and Load Cell

Linear actuator: 900 lbf capacity, operating at 550 lbf = 61% of rated.

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Buckling at 6 in stroke, 900 lbf: P_cr >> load, FoS ~170
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Load cell: S-beam, 1,000 lbf rated; 1:1 lever ratio means it reads wheel force directly.

8. Summary of Factors of Safety

Component	Loading	FoS Target	FoS Achieved
Drive roller tube (aluminum)	Bending	>= 2.0	37.6
Drive roller dead axle (steel)	Bending	>= 2.0	20.1
Drive roller bearings	Dynamic load	>= 2.0	5.5
Chain (#35)	Peak tension	>= 2.0	2.5
Motor torque	Continuous	>= 1.5	1.58
Belt deck UHMW	Compression	>= 2.0	93
Belt deck cross tubes (realistic)	Bending	>= 2.0	18.2
Belt deck cross tubes (worst case)	Bending	>= 2.0	5.6
Lever arm tube (single lever)	Bending	>= 2.0	3.6
4-bar arm (2x2x1/4)	Axial	>= 2.0	>100
Pivot shaft	Bending	>= 2.0	2.4
Pivot bushings	Bearing pressure	>= 1.5	1.67
Linear actuator rod	Buckling	>= 2.0	~170

All components meet their factor-of-safety targets. Structural members (roller, axle, deck) are conservatively overbuilt. The tightest margins are motor torque (1.58, intermittent duty) and pivot bushing bearing pressure (1.67, worst-case load only). These analytical results form the basis for the FEA validation that follows.